**Министерство науки и высшего образования Российской Федерации**

федеральное государственное автономное образовательное учреждение

высшего образования

**«НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ**

**ТОМСКИЙ ПОЛИТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ»**

Инженерная школа природных ресурсов

Направление подготовки Химическая технология

Отделение химической инженерии

**Численные методы решения систем обыкновенных дифференциальных уравнений**

Выполнил студент гр. 2Д93 Ю.Н. Милованова

(Подпись)

12.06. 2020 г.

Отчет принят:

Преподаватель

доцент ОХИ ИШПР, к.т.н. В.А. Чузлов

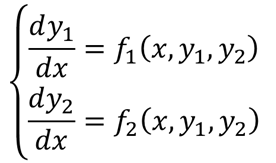
(Подпись)

\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_ 2020 г.

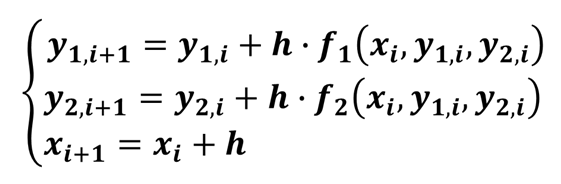
Томск 2020 г.

**Цель работы:** научиться реализовывать программы с использованием численных методов решения систем обыкновенных дифференциальных уравнений.

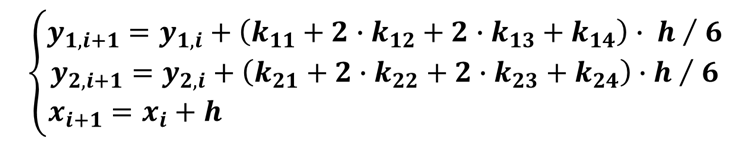
**Теоретическая часть**

Дифференциальное уравнение — уравнение, в которое входят производные функции, и может входить сама функция, независимая переменная и параметры. Имеет вид . Для решения таких уравнений можно воспользоваться следующими методами:

* Метод Эйлера – относительно не точный.

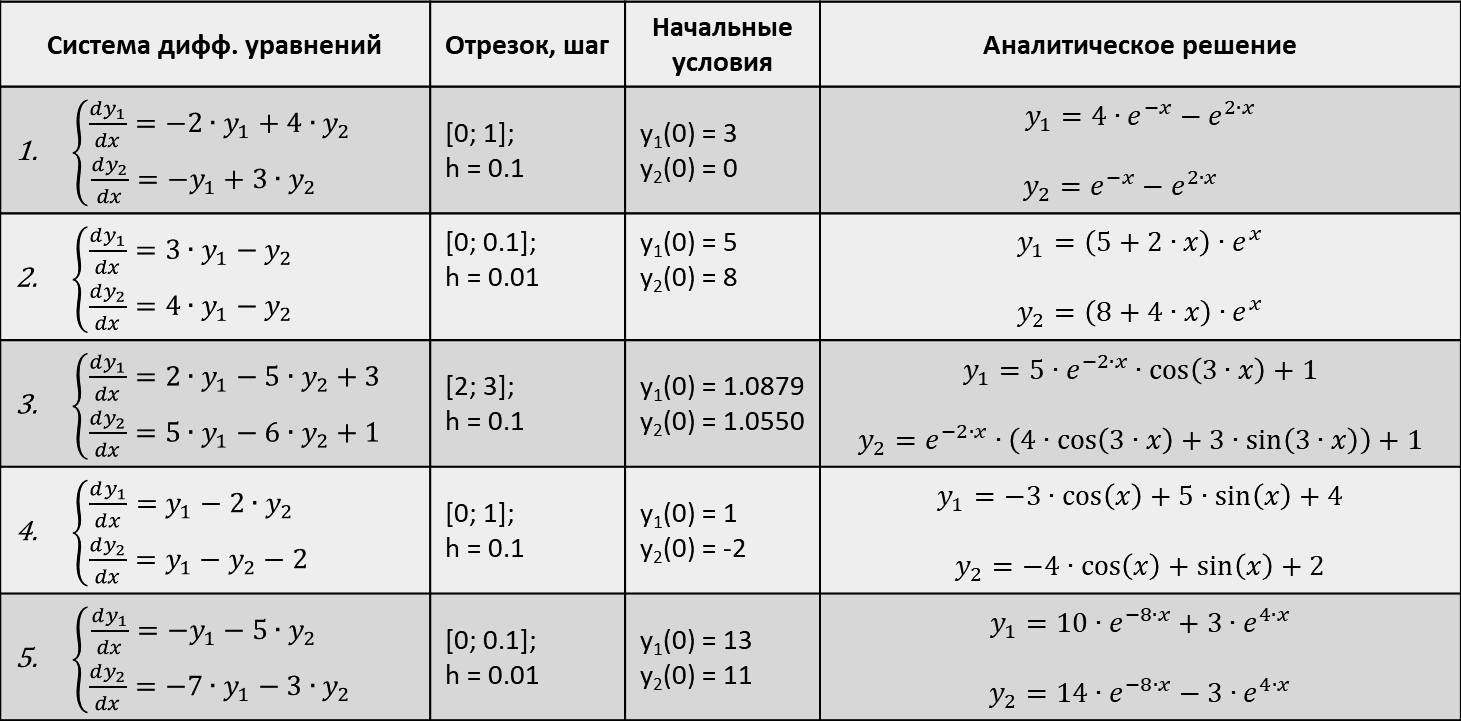


* Метод Рунге-Кутты – занимает много больше объема вычислений по сравнению с методом Эйлера, но значительно точнее предыдущего.



**Практическая часть**

**Исходные данные:**

****

**Задание:**

Решить систему дифференциальных уравнений методами Эйлера и Рунге-Кутты. Определить погрешность расчетного значения переменной y для каждого из методов, использовав формулу:

где - расчетное значение, - значение, полученное из аналитического решения.

**Программная реализация:**

Метод Эйлера. Пример 1.

**Program** lb131e;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a(x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := -2\*y1 + 4\*y2;

result[1] := -y1 + 3\*y2

**end**;

**function** al (x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 4\*exp(-x\*ln(e)) - exp(2\*x\*ln(e));

result[1] := exp(-x\*ln(e)) - exp(2\*x\*ln(e));

**end**;

**function** eyler(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

f: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

f := a(x, y1, y2);

y1 := y1 + h \* f[0];

y2 := y2 + h \* f[1];

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:10:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(eyler(0, 1, 0.1, 3, 0)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 3.0000 0.0000 3.0000 0.0000 0.0000 0.0000

0.1000 2.4000 -0.3000 2.3979 -0.3166 0.0856 5.2328

0.2000 1.8000 -0.6300 1.7831 -0.6731 0.9479 6.4024

0.3000 1.1880 -0.9990 1.1412 -1.0813 4.1051 7.6113

0.4000 0.5508 -1.4175 0.4557 -1.5552 20.8586 8.8554

0.5000 -0.1264 -1.8978 -0.2922 -2.1118 56.7496 10.1300

0.6000 -0.8602 -2.4545 -1.1249 -2.7713 23.5272 11.4301

0.7000 -1.6700 -3.1049 -2.0689 -3.5586 19.2795 12.7502

0.8000 -2.5779 -3.8693 -3.1557 -4.5037 18.3086 14.0852

0.9000 -3.6101 -4.7724 -4.4234 -5.6431 18.3858 15.4298

1.0000 -4.7970 -5.8431 -5.9175 -7.0212 18.9355 16.7795

Метод Рунге-Кутты. Пример 1.

**Program** lb131r;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a (x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := -2\*y1 + 4\*y2;

result[1] := -y1 + 3\*y2

**end**;

**function** al(x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 4\*exp(-x\*ln(e)) - exp(2\*x\*ln(e));

result[1] := exp(-x\*ln(e)) - exp(2\*x\*ln(e));

**end**;

**function** rk(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

k1, k2, k3, k4: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

k1 := a(x, y1, y2);

k2 := a(x + h / 2, y1 + k1[0] \* h / 2, y2 + k1[1] \* h / 2);

k3 := a(x + h / 2, y1 + k2[0] \* h / 2, y2 + k2[1] \* h / 2);

k4 := a(x + h, y1 + k3[0] \* h, y2 + k3[1] \* h);

y1 := y1 + h / 6 \* (k1[0] + 2 \* k2[0] + 2 \* k3[0] + k4[0]);

y2 := y2 + h / 6 \* (k1[1] + 2 \* k2[1] + 2 \* k3[1] + k4[1]);

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:10:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(rk(0, 1, 0.1, 3, 0)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 3.0000 0.0000 3.0000 0.0000 0.0000 0.0000

0.1000 2.3980 -0.3166 2.3979 -0.3166 0.0001 0.0009

0.2000 1.7831 -0.6731 1.7831 -0.6731 0.0004 0.0010

0.3000 1.1412 -1.0813 1.1412 -1.0813 0.0012 0.0012

0.4000 0.4558 -1.5552 0.4557 -1.5552 0.0046 0.0013

0.5000 -0.2921 -2.1117 -0.2922 -2.1118 0.0109 0.0015

0.6000 -1.1248 -2.7713 -1.1249 -2.7713 0.0041 0.0016

0.7000 -2.0688 -3.5586 -2.0689 -3.5586 0.0032 0.0018

0.8000 -3.1556 -4.5036 -3.1557 -4.5037 0.0029 0.0020

0.9000 -4.4232 -5.6430 -4.4234 -5.6431 0.0028 0.0022

1.0000 -5.9174 -7.0210 -5.9175 -7.0212 0.0028 0.0024

Метод Эйлера. Пример 2.

**Program** lb132e;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a(x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := 3\*y1 - y2;

result[1] := 4\*y1 - y2

**end**;

**function** al (x: real): arr;

**begin**

SetLength(result, 2);

result[0] := (5 + 2\*x)\*exp(x\*ln(e));

result[1] := (8 + 4\*x)\*exp(x\*ln(e));

**end**;

**function** eyler(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

f: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

f := a(x, y1, y2);

y1 := y1 + h \* f[0];

y2 := y2 + h \* f[1];

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(eyler(0, 0.1, 0.01, 5, 8)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 5.0000 8.0000 5.0000 8.0000 0.0000 0.0000

0.0100 5.0700 8.1200 5.0705 8.1208 0.0089 0.0099

0.0200 5.1409 8.2416 5.1418 8.2432 0.0178 0.0197

0.0300 5.2127 8.3648 5.2141 8.3673 0.0266 0.0295

0.0400 5.2854 8.4897 5.2873 8.4930 0.0355 0.0393

0.0500 5.3591 8.6162 5.3615 8.6204 0.0442 0.0490

0.0600 5.4337 8.7444 5.4366 8.7495 0.0530 0.0586

0.0700 5.5093 8.8743 5.5127 8.8804 0.0617 0.0682

0.0800 5.5858 9.0059 5.5898 9.0129 0.0704 0.0778

0.0900 5.6633 9.1393 5.6678 9.1473 0.0791 0.0873

0.1000 5.7418 9.2745 5.7469 9.2834 0.0877 0.0968

Метод Рунге-Кутты. Пример 2.

**Program** lb132r;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a (x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := -2 \* y1 + 4 \* y2;

result[1] := -y1 + 3 \* y2

**end**;

**function** al(x: real): arr;

**begin**

SetLength(result, 2);

result[0] := (5 + 2 \* x) \* exp(x \* ln(e));

result[1] := (8 + 4 \* x) \* exp(x \* ln(e));

**end**;

**function** rk(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

k1, k2, k3, k4: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

k1 := a(x, y1, y2);

k2 := a(x + h / 2, y1 + k1[0] \* h / 2, y2 + k1[1] \* h / 2);

k3 := a(x + h / 2, y1 + k2[0] \* h / 2, y2 + k2[1] \* h / 2);

k4 := a(x + h, y1 + k3[0] \* h, y2 + k3[1] \* h);

y1 := y1 + h / 6 \* (k1[0] + 2 \* k2[0] + 2 \* k3[0] + k4[0]);

y2 := y2 + h / 6 \* (k1[1] + 2 \* k2[1] + 2 \* k3[1] + k4[1]);

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(rk(0, 0.1, 0.01, 5, 8)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 5.0000 8.0000 5.0000 8.0000 0.0000 0.0000

0.0100 5.2216 8.1918 5.0705 8.1208 2.9812 0.8738

0.0200 5.4465 8.3871 5.1418 8.2432 5.9257 1.7453

0.0300 5.6747 8.5861 5.2141 8.3673 8.8346 2.6149

0.0400 5.9064 8.7888 5.2873 8.4930 11.7093 3.4826

0.0500 6.1416 8.9953 5.3615 8.6204 14.5508 4.3488

0.0600 6.3804 9.2057 5.4366 8.7495 17.3603 5.2137

0.0700 6.6229 9.4201 5.5127 8.8804 20.1389 6.0775

0.0800 6.8691 9.6385 5.5898 9.0129 22.8878 6.9404

0.0900 7.1192 9.8610 5.6678 9.1473 25.6079 7.8026

0.1000 7.3733 10.0878 5.7469 9.2834 28.3003 8.6644

Метод Эйлера. Пример 3.

**Program** lb133e;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a(x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := 2\*y1 - 5\*y2 + 3;

result[1] := 5\*y1 - 6\*y2 + 1;

**end**;

**function** al (x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 5\*exp(-2\*x\*ln(e)) \* cos(3\*x) + 1;

result[1] := exp(-2\*x\*ln(e))\*(4\*cos(3\*x) + 3\*sin(3\*x)) + 1;

**end**;

**function** eyler(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

f: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

f := a(x, y1, y2);

y1 := y1 + h \* f[0];

y2 := y2 + h \* f[1];

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(eyler( 2, 3, 0.1, 1.0879, 1.0550)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

2.0000 1.0879 1.0550 1.0879 1.0550 0.0000 0.0000

2.1000 1.0780 1.0660 1.0750 1.0607 0.2803 0.4921

2.2000 1.0606 1.0654 1.0583 1.0581 0.2144 0.6833

2.3000 1.0400 1.0564 1.0410 1.0502 0.0924 0.5910

2.4000 1.0198 1.0426 1.0250 1.0396 0.5086 0.2863

2.5000 1.0025 1.0269 1.0117 1.0283 0.9088 0.1317

2.6000 0.9895 1.0120 1.0015 1.0177 1.1964 0.5595

2.7000 0.9814 0.9996 0.9945 1.0087 1.3175 0.9100

2.8000 0.9779 0.9905 0.9904 1.0018 1.2624 1.1256

2.9000 0.9782 0.9852 0.9887 0.9970 1.0574 1.1834

3.0000 0.9813 0.9832 0.9887 0.9940 0.7515 1.0926

Метод Рунге-Кутты. Пример 3.

**Program** lb133r;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a (x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := 2 \* y1 - 5 \* y2 + 3;

result[1] := 5 \* y1 - 6 \* y2 + 1;

**end**;

**function** al(x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 5 \* exp(-2 \* x \* ln(e)) \* cos(3 \* x) + 1;

result[1] := exp(-2 \* x \* ln(e)) \* (4 \* cos(3 \* x) + 3 \* sin(3 \* x)) + 1;

**end**;

**function** rk(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

k1, k2, k3, k4: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

k1 := a(x, y1, y2);

k2 := a(x + h / 2, y1 + k1[0] \* h / 2, y2 + k1[1] \* h / 2);

k3 := a(x + h / 2, y1 + k2[0] \* h / 2, y2 + k2[1] \* h / 2);

k4 := a(x + h, y1 + k3[0] \* h, y2 + k3[1] \* h);

y1 := y1 + h / 6 \* (k1[0] + 2 \* k2[0] + 2 \* k3[0] + k4[0]);

y2 := y2 + h / 6 \* (k1[1] + 2 \* k2[1] + 2 \* k3[1] + k4[1]);

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(rk(2, 3, 0.1, 1.0879, 1.0550)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

2.0000 1.0879 1.0550 1.0879 1.0550 0.0000 0.0000

2.1000 1.0749 1.0607 1.0750 1.0607 0.0033 0.0004

2.2000 1.0583 1.0581 1.0583 1.0581 0.0035 0.0013

2.3000 1.0410 1.0502 1.0410 1.0502 0.0034 0.0019

2.4000 1.0250 1.0396 1.0250 1.0396 0.0032 0.0022

2.5000 1.0117 1.0283 1.0117 1.0283 0.0028 0.0023

2.6000 1.0015 1.0177 1.0015 1.0177 0.0023 0.0023

2.7000 0.9945 1.0087 0.9945 1.0087 0.0017 0.0020

2.8000 0.9904 1.0018 0.9904 1.0018 0.0012 0.0017

2.9000 0.9887 0.9969 0.9887 0.9970 0.0007 0.0014

3.0000 0.9887 0.9940 0.9887 0.9940 0.0003 0.0010

Метод Эйлера. Пример 4.

**Program** lb134e;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a(x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := y1 - 2\*y2;

result[1] := y1 - y2 - 2;

**end**;

**function** al (x: real): arr;

**begin**

SetLength(result, 2);

result[0] := -3\*cos(x) + 5\*sin(x) + 4;

result[1] := -4\*cos(x) + sin(x) + 2;

**end**;

**function** eyler(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

f: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

f := a(x, y1, y2);

y1 := y1 + h \* f[0];

y2 := y2 + h \* f[1];

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(eyler( 0, 1, 0.1, 1, -2)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 1.0000 -2.0000 1.0000 -2.0000 0.0000 0.0000

0.1000 1.5000 -1.9000 1.5142 -1.8802 0.9348 1.0540

0.2000 2.0300 -1.7600 2.0531 -1.7216 1.1274 2.2307

0.3000 2.5850 -1.5810 2.6116 -1.5258 1.0182 3.6160

0.4000 3.1597 -1.3644 3.1839 -1.2948 0.7603 5.3733

0.5000 3.7486 -1.1120 3.7644 -1.0309 0.4205 7.8654

0.6000 4.3458 -0.8259 4.3472 -0.7367 0.0323 12.1129

0.7000 4.9456 -0.5088 4.9266 -0.4152 0.3858 22.5487

0.8000 5.5419 -0.1633 5.4967 -0.0695 0.8227 135.1045

0.9000 6.1287 0.2072 6.0518 0.2969 1.2712 30.2118

1.0000 6.7002 0.5993 6.5864 0.6803 1.7266 11.8948

Метод Рунге-Кутты. Пример 4.

**Program** lb134r;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a (x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := y1 - 2\*y2;

result[1] := y1 - y2 - 2;

**end**;

**function** al(x: real): arr;

**begin**

SetLength(result, 2);

result[0] := -3\*cos(x) + 5\*sin(x) + 4;

result[1] := -4\*cos(x) + sin(x) + 2;

**end**;

**function** rk(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

k1, k2, k3, k4: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

k1 := a(x, y1, y2);

k2 := a(x + h / 2, y1 + k1[0] \* h / 2, y2 + k1[1] \* h / 2);

k3 := a(x + h / 2, y1 + k2[0] \* h / 2, y2 + k2[1] \* h / 2);

k4 := a(x + h, y1 + k3[0] \* h, y2 + k3[1] \* h);

y1 := y1 + h / 6 \* (k1[0] + 2 \* k2[0] + 2 \* k3[0] + k4[0]);

y2 := y2 + h / 6 \* (k1[1] + 2 \* k2[1] + 2 \* k3[1] + k4[1]);

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(rk(0, 1, 0.1, 1, -2)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 1.0000 -2.0000 1.0000 -2.0000 0.0000 0.0000

0.1000 1.5142 -1.8802 1.5142 -1.8802 0.0000 0.0000

0.2000 2.0531 -1.7216 2.0531 -1.7216 0.0000 0.0000

0.3000 2.6116 -1.5258 2.6116 -1.5258 0.0001 0.0000

0.4000 3.1839 -1.2948 3.1839 -1.2948 0.0001 0.0001

0.5000 3.7644 -1.0309 3.7644 -1.0309 0.0001 0.0001

0.6000 4.3472 -0.7367 4.3472 -0.7367 0.0001 0.0002

0.7000 4.9266 -0.4152 4.9266 -0.4152 0.0001 0.0004

0.8000 5.4967 -0.0695 5.4967 -0.0695 0.0001 0.0032

0.9000 6.0518 0.2969 6.0518 0.2969 0.0001 0.0009

1.0000 6.5864 0.6803 6.5864 0.6803 0.0001 0.0005

Метод Эйлера. Пример 5.

**Program** lb135e;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a(x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := -y1 - 5\*y2;

result[1] := -7\*y1 - 3\*y2;

**end**;

**function** al (x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 10\*exp(-8\*x\*ln(e)) + 3\*exp(4\*x\*ln(e));

result[1] := 14\*exp(-8\*x\*ln(e)) - 3\*exp(4\*x\*ln(e));

**end**;

**function** eyler(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

f: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

f := a(x, y1, y2);

y1 := y1 + h \* f[0];

y2 := y2 + h \* f[1];

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(eyler( 0, 0.1, 0.01, 13, 11)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 13.0000 11.0000 13.0000 11.0000 0.0000 0.0000

0.0100 12.3200 9.7600 12.3536 9.8012 0.2720 0.4203

0.0200 11.7088 8.6048 11.7713 8.6802 0.5309 0.8681

0.0300 11.1615 7.5270 11.2488 7.6303 0.7761 1.3533

0.0400 10.6735 6.5199 10.7820 6.6456 1.0065 1.8904

0.0500 10.2408 5.5772 10.3674 5.7203 1.2215 2.5014

0.0600 9.8595 4.6930 10.0016 4.8492 1.4205 3.2213

0.0700 9.5263 3.8621 9.6815 4.0275 1.6033 4.1087

0.0800 9.2379 3.0794 9.4043 3.2507 1.7695 5.2713

0.0900 8.9915 2.3403 9.1675 2.5145 1.9194 6.9285

0.1000 8.7846 1.6407 8.9688 1.8151 2.0532 9.6095

Метод Рунге-Кутты. Пример 5.

**Program** lb135r;

**type**

matrix = **array of array of** real;

arr = **array of** real;

**function** a (x, y1, y2: real): arr;

**begin**

SetLength(result, 2);

result[0] := -y1 - 5\*y2;

result[1] := -7\*y1 - 3\*y2;

**end**;

**function** al(x: real): arr;

**begin**

SetLength(result, 2);

result[0] := 10\*exp(-8\*x\*ln(e)) + 3\*exp(4\*x\*ln(e));

result[1] := 14\*exp(-8\*x\*ln(e)) - 3\*exp(4\*x\*ln(e));

**end**;

**function** rk(start\_x, stop\_x, h, start\_y1, start\_y2: real): matrix;

**var**

i: integer;

x, y1, y2: real;

k1, k2, k3, k4: arr;

**begin**

SetLength(result, Trunc((stop\_x - start\_x) / h) + 1);

**for** i := 0 **to** High(result) **do**

SetLength(result[i], 3);

x := start\_x;

y1 := start\_y1;

y2 := start\_y2;

**for** i := 0 **to** High(result) **do**

**begin**

result[i, 0] := x;

result[i, 1] := y1;

result[i, 2] := y2;

k1 := a(x, y1, y2);

k2 := a(x + h / 2, y1 + k1[0] \* h / 2, y2 + k1[1] \* h / 2);

k3 := a(x + h / 2, y1 + k2[0] \* h / 2, y2 + k2[1] \* h / 2);

k4 := a(x + h, y1 + k3[0] \* h, y2 + k3[1] \* h);

y1 := y1 + h / 6 \* (k1[0] + 2 \* k2[0] + 2 \* k3[0] + k4[0]);

y2 := y2 + h / 6 \* (k1[1] + 2 \* k2[1] + 2 \* k3[1] + k4[1]);

x := x + h

**end**;

**end**;

**function** func(y: matrix): matrix;

**var**

i, j: integer;

x: real;

f1: arr;

**begin**

SetLength(result, Length (y));

**for** i := 0 **to** High(result) **do**

SetLength(result[i], {4} 7);

**for** i := 0 **to** High(result) **do**

**begin**

f1 := al(y[i, 0]);

result[i, 3] := f1[0];

result[i, 4] := f1[1];

**if** i = 0 **then**

**begin**

result[i, 5] := 0;

result[i, 6] := 0;

**end**

**else**

**begin**

result[i, 5] := abs((y[i, 1] - f1[0])/f1[0]) \* 100;

result[i, 6]:= abs((y[i, 2] - f1[1])/f1[1]) \* 100;

**end**;

**for** j := 0 **to** 2 **do**

result [i, j] := y[i, j];

**end**;

**end**;

**procedure** print\_results(res: matrix);

**var**

i, j: integer;

**begin**

**for** i := 0 **to** High(res) **do**

**begin**

**for** j := 0 **to** High(res[i]) **do**

write(res[i, j]:9:4);

writeln

**end**;

**end**;

**begin**

writeln(' x ',' y1 ', ' y2 ', ' ya1 ', ' ya2 ',

' d1', ' d2 ');

print\_results(func(rk(0, 0.1, 0.01, 13, 11)))

**end**.

**Ответы:**

x y1 y2 ya1 ya2 d1 d2

0.0000 13.0000 11.0000 13.0000 11.0000 0.0000 0.0000

0.0100 12.3536 9.8012 12.3536 9.8012 0.0000 0.0000

0.0200 11.7713 8.6802 11.7713 8.6802 0.0000 0.0000

0.0300 11.2488 7.6303 11.2488 7.6303 0.0000 0.0000

0.0400 10.7820 6.6456 10.7820 6.6456 0.0000 0.0000

0.0500 10.3674 5.7203 10.3674 5.7203 0.0000 0.0000

0.0600 10.0016 4.8492 10.0016 4.8492 0.0000 0.0000

0.0700 9.6815 4.0275 9.6815 4.0275 0.0000 0.0000

0.0800 9.4043 3.2507 9.4043 3.2507 0.0000 0.0001

0.0900 9.1675 2.5145 9.1675 2.5145 0.0000 0.0001

0.1000 8.9688 1.8151 8.9688 1.8151 0.0000 0.0001

**Выводы**

В ходе лабораторной работы были изучены методы численного решения систем обыкновенных дифференциальных уравнений - метод Эйлера и метод Рунге-Кутты. Исходя из определений погрешности, был сделан вывод о более высокой точности метода Рунге-Кутты.